

Closing Tues: HW 14.2 (part 2)
Closing Thur: HW 14.3/4 (last HW)

Final: Sat, March 10, 5:00-7:50pm,

Final Room is based on quiz section

For BC/BD, AC/AD: PAA A102

For AA/AB, BB: PAA A118

For BA: PAA A110

Entry Task:

Find all critical points of

$$z = f(x, y) = 2x^4 + y^2 - 4xy + 1$$

$$\frac{\partial z}{\partial x} = 8x^3 - 4y \stackrel{?}{=} 0 \Rightarrow 8x^3 = 4y \Rightarrow y = 2x^3$$

$$\frac{\partial z}{\partial y} = 2y - 4x \stackrel{?}{=} 0$$

COMBINE!!!

$$2(2x^3) - 4x \stackrel{?}{=} 0$$

$$4x^3 - 4x \stackrel{?}{=} 0$$

$$4x(x^2 - 1) = 0$$

$$x = 0 \quad \text{or} \quad x^2 = 1$$

$\swarrow \quad \searrow$
 $x = -1 \quad x = 1$

THREE POSSIBILITIES

$$x = 0 \Rightarrow y = 0 \Rightarrow (0, 0)$$

$$x = -1 \Rightarrow y = -2 \Rightarrow (-1, -2)$$

$$x = 1 \Rightarrow y = 2 \Rightarrow (1, 2)$$

You do (HW Problem 14.2/5)

$$z = -6x^2 + 2x - 4y^2 - 3y + 8xy + 30$$

$= f(x,y)$

a. Find the critical point.

$$\frac{\partial z}{\partial x} = -12x + 2 + 8y \stackrel{?}{=} 0$$

$$8y = 12x - 2$$

$$y = \frac{12x - 2}{8}$$

$$\frac{\partial z}{\partial y} = -8y - 3 + 8x \stackrel{?}{=} 0$$

CASING!!

$$-8\left(\frac{12x - 2}{8}\right) - 3 + 8x \stackrel{?}{=} 0$$

$$-12x + 2 - 3 + 8x = 0$$

$$-1 = 4x$$

$$x = -\frac{1}{4} \Rightarrow y = \frac{-3 - 2}{8} = -\frac{5}{8}$$

b. Find the largest and smallest values of $f(2,y)$ on the interval $y=-4$ to $y=0$.

$$f(2,y) = -24 + 2 - 4y^2 - 3y + 16y + 30$$

$$= -4y^2 + 13y + 10$$

$$f_y(2,y) = -8y + 13 = 0 \Rightarrow y = \frac{13}{8} = 1.625$$

$$f(2,-4) = -4(-4)^2 + 13(-4) + 10 = -106$$

$$f(2,0) = -4(0) + 13(0) + 10 = 10$$

c. Suppose $(x,y) = (-6,-7)$.

A small increase in x will lead to a **LARGER**/SMALLER increase in z than a small increase in y . (circle either larger or smaller)

$$f_x(-6,-7) = -12(-6) + 2 + 8(-7) = 18$$

$$f_y(-6,-7) = -8(-7) - 3 + 8(-6) = 5$$

d. Which is steepest at $x = 1$?

$f(x,4)$, $f(x,6)$, $f(x,8)$ or $f(x,10)$

$$f(x,4) = -6x^2 + 2x - 64 - 12 + 32x + 30$$

$$f_x(1,4) = -12(1) + 2 + 8(4) = 22$$

$$f_x(1,6) = -12(1) + 2 + 8(6) = 38$$

$$f_x(1,8) = -12(1) + 2 + 8(8) = 54$$

$$f_x(1,10) = -12(1) + 2 + 8(10) = 70$$

$\frac{\partial z}{\partial x}$



OUTSIDE RANGE

Example (From HW 14.2/6)

Find the critical point for

$$f(x, y) = 12 + xy + \frac{27}{x} + \frac{8}{y} = 12 + xy + 27x^{-1} + 8y^{-1}$$

$$f_x = y - 27x^{-2} = y - \frac{27}{x^2} = 0$$

$$y = \frac{27}{x^2} \quad \text{COMBINE!!!}$$

$$f_y = x - 8y^{-2} = x - \frac{8}{y^2} = 0$$

$$xy^2 - 8 = 0$$

$$x \left(\frac{27}{x^2} \right)^2 - 8 = 0$$

$$x \cdot \frac{729}{x^4} - 8 = 0$$

$$\frac{729}{x^3} - 8 = 0$$

$$\Rightarrow 729 - 8x^3 = 0$$

$$\Rightarrow 729 = 8x^3$$

$$\Rightarrow x^3 = \frac{729}{8}$$

$$\Rightarrow x = \left(\frac{729}{8} \right)^{1/3}$$

$$x = 4.5$$

$$y = \frac{27}{(4.5)^2} = 1.33$$

More Applications

Cost Breakdown (14.3/1-2)

Suppose the cost to produce ONE item is given by:

$$C(x, y) = 3x^2 + 4y^2 + 5xy + 10,$$

where

x = cost for 1 hour of labor, and

y = cost for 1 pound of materials.

$$C_x = 6x + 5y$$

$\frac{\text{cost}}{\text{labor hours}}$

$$C_y = 8y + 5x$$

$\frac{\text{cost}}{\text{pound}}$

Question:

The current hourly rate for labor is \$20 and material is \$55 per pound.

How will a \$1 per hour raise for labor affect the cost to produce 1 item?

$$C_x(20, 55) = 6(20) + 5(55) = 395$$

Marginal Productivity (14.3/5-6)

Suppose that the number of crates of a particular fruit produced is

$$z = \frac{9xy - 0.0002x^2 - 5y}{0.03x + 4y} \leftarrow N$$

where

x = number of hours of labor, and

y = number of acres of the crop.

Find the marginal productivity of the number of hours of labor when

$x = 100$ and $y = 200$.

Interpret your answer

$$\frac{dz}{dx} = \frac{\text{crates}}{\text{labor hours}} = \text{marginal productivity}$$

$$N = 9xy - 0.0002x^2 - 5y$$

$$N' = 9y - 0.0004x$$

$$D = 0.03x + 4y$$

$$D' = 0.03$$

$$\frac{dz}{dx} = \frac{(0.03x + 4y)(9y - 0.0004x) - (9xy - 0.0002x^2 - 5y)(0.03)}{(0.03x + 4y)^2}$$

$$\text{at } x = 100, y = 200$$

$$\frac{dz}{dx} = \frac{(3 + 800)(1800 - 0.04) - (9(100)(200) - 0.0002(100)^2 - 5(200))(0.03)}{(3 + 800)^2}$$

$$\approx 2.23322 \frac{\text{crates}}{\text{labor hours}}$$